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Entanglement dynamics and decoherence of three-qubit system in a fermionic environment

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Abstract

We study the entanglement dynamics of the three-qubit system in a symmetry-broken environment consisting of a fermionic bath. Decoherence induced by the bath is analysed. We find that the entanglement of states will decrease or remain unchanged under the system–bath interaction. The class of decoherence-free states of the three-qubit system of our model has been found out.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Entanglement is one of the inherent features of quantum mechanics [1], and it also plays an important role in quantum information processing (QIP). As such, entanglement is worth studying in every respect. The dynamics of entanglement is an interesting problem. To some extent, the dynamics of entanglement is the time evolution of entanglement measure [2–5]. More recently, much effort has been devoted to thermal entanglement [6], i.e. quantifying entanglement produced by a thermal bath with the temperature at which the equilibrium is reached. In this context the entanglement dynamics is studied mainly based on a master equation and Markovian approximation [7]. In QIP, decoherence is another essential problem that deserves some attention. Generally, decoherence is used to estimate the deviation from an ideal state.

Multi-qubit systems are of interest to be investigated both theoretically and experimentally. In recent years, a lot of achievements have been obtained on the application of three-qubit states to QIP [8–11], so understanding of entanglement and decoherence of the three-qubit system under environment is important for us in QIP. Entanglement dynamics and decoherence have been studied in the frame of various models [12–15]. In [15], the authors have considered

a transverse Ising model (TIM) and studied the concurrence of a bipartite system under a symmetry-broken environment. A model such as the long-range Ising model introduced in [14] (IM) has two features. Firstly, the bath density is assumed to be a thermal distribution, that is $\rho_b = (e^{-H_b/T})/Z$, with T the bath temperature multiplied by the Boltzmann constant, and $Z = \text{Tr}(e^{-H_b/T})$ the partition function. Secondly, the bath Hamiltonian H_b needs to be a symmetry breakable one, that is endowed with phase transition in the degrees of freedom that provide the coupling with the system. In [15], the authors have an analysis by adding a transverse field to the thermal bath, investigating eventually a transverse Ising model bath Hamiltonian (TIM). Our work is developed based on the model studied in [15].

In this paper, we make an extension of the model from the two-qubit system to the three-qubit system, keeping the symmetry-broken environment unchanged. By using the mean field approximation (MF), we study the entanglement dynamics and decoherence of the three-qubit system in the environment. Such an extension involves the related entanglement measure and decoherence definition; we will employ the entanglement measure [16] and linear entropy [17] as a measure for decoherence to investigate entanglement dynamics and decoherence.

The paper is organized as follows: in section 2, we make an extension of the model. We give our main results in section 3. Finally, in section 4 we give the conclusion.

2. The model and some notation

We consider a general model consisting of a system and a thermal bath, and the model is described by the Hamiltonian $H = H_s + H_{sb} + H_b$, where H_s , H_b and H_{sb} denote the Hamiltonian of the system, bath and system–bath interaction respectively. One is often interested in the evolution of a reduced system density matrix by which one can know the entanglement dynamics and decoherence. While under the precondition that the Hamiltonian is time independent, the whole density matrix evolves

$$\rho(t) = e^{-iHt} \rho(0) e^{iHt} \quad (1)$$

where we assume that the initial density $\rho(0)$ is separable between the system and bath, i.e. $\rho(0) = \rho_s(0) \otimes \rho_b$. The bath density matrix is assumed to be $\rho_b = e^{(-H_b/T)}/Z$. We are interested in the reduced system density matrix $\rho_s(t)$ which can be obtained by tracing out the degree of freedom of the thermal bath,

$$\rho_s(t) = \text{Tr}_b(\rho(t)). \quad (2)$$

The Hamiltonian form is the deterministic factor in the analysis of entanglement dynamics and decoherence.

2.1. Hamiltonian form

We extend the system Hamiltonian of the TIM model in [15] from two-qubit to three-qubit and assume that the system qubits, labelled as A, B and C, interact with each other and with the spin bath,

$$H_s = -\xi_0 (S_A^z S_B^z + S_B^z S_C^z + S_A^z S_C^z) \quad (3)$$

$$H_{sb} = -\frac{J_0}{\sqrt{N}} (S_A^z + S_B^z + S_C^z) \sum_k S_k^z \quad (4)$$

$$H_b = -\omega \sum_k S_k^x - \frac{J}{N} \sum_{i,k} S_i^z S_k^z \quad (5)$$

where ξ_0 is the coupling constant between qubits, J_0, J are exchange coupling constants and ω is the strength of the transverse field; all these are non-negative constants. The indices of the sum run from 1 to N .

It is not difficult to deal with the above Hamiltonian except for H_b because of the nonlinear term. But by using the mean field approximation (MF), we can do it easily. Details concerning the MF are explained in [18]. We will use the MF directly:

$$H_b^{\text{mf}} = -\omega \sum_k S_k^x - 2Jm \sum_k S_k^z + m^2 JN \tag{6}$$

where m is the order parameter of the phase transition. Its absolute value ranges from 0 to $\frac{1}{2}$ as long as the temperature ranges from the critical value $T_C = J/2$ to 0. Using the mean field approximation, we obtain a Curie–Weiss equation in [18],

$$\frac{\Theta}{J} = \tanh \frac{\Theta}{2T} \tag{7}$$

and the quantity $\Theta = \pm\sqrt{\omega^2 + 4m^2 J^2}$. For the H_b z symmetry, so every thing will remain true with substitution $m \rightarrow -m$ or $\Theta \rightarrow -\Theta$. We will take the positive values of m and Θ in the following text for convenience.

2.2. Entanglement measure definition for the three-qubit system

There are several kinds of definitions of entanglement measure of the three-qubit system. In this paper, we will employ that proposed by Vidal in [16] as an entanglement measure. Based on Pere’s criterion for separability [19], given a system state ρ_s either pure or mixed, Vidal *et al* introduce a computable entanglement measure defined as the *negativity* to quantify the degree to which $\rho_s^{T_A}$ fails to be positive,

$$N(\rho_s) = \frac{\|\rho_s^{T_A}\| - 1}{2} \tag{8}$$

where $\rho_s^{T_A}$ is the partial transpose of ρ_s with respect to party A and $\|\rho_s^{T_A}\|$ means the sum of absolute values of eigenvalues of $\rho_s^{T_A}$, that is

$$\|\rho_s^{T_A}\| = \sum_i |\lambda_i| = \sum_i \lambda_i + 2 \sum_j |\lambda_j|$$

where λ_i is the eigenvalue of $\rho_s^{T_A}$ and λ_j is the negative eigenvalue among λ_i , with the known relation

$$\sum_i \lambda_i = \text{Tr}(\rho_s^{T_A}) = 1.$$

Then we know that $N(\rho_s)$ corresponds to the absolute value of the sum of negative eigenvalues of $\rho_s^{T_A}$ [20], and vanishes for unentangled states.

Vidal has proved that $N(\rho_s)$ does not increase under *LOCC* (local operation and classical communication), i.e. it is an entanglement monotone. For the case of three-party state ρ_{ABC} , we can compute the sum of negative eigenvalues of $\rho_{ABC}^{T_C}$ which is the partial transpose of ρ_{ABC} with respect to party C, $N_{(AB)-C}(\rho_{ABC})$ is automatically an entanglement monotone, which quantifies the strength of quantum correlation between party C and the joint of two parties (AB). Similarly, the *negativities* $N_{(AC)-B}(\rho_{ABC})$ and $N_{(BC)-A}(\rho_{ABC})$ are two other monotonic functions under *LOCC* with analogous meaning.

When the party C is separable from the two parties (AB), we can also consider the entanglement properties of two-party reduced density matrices $\rho_{AB} = \text{Tr}_C(\rho_{ABC})$. The

negativity of the reduced system density matrix ρ_{AB} , $N_{A-B;C}$, can be used to quantify the residual entanglement, and $N_{A-C;B}$, $N_{B-C;A}$ are of analogous meaning. Thus, we have six computable functions to quantify the entanglement of any tripartite system including the system we consider here.

2.3. Measure for decoherence

We usually are interested in the derivation from an ideal, usually pure state with time changing, i.e. the dynamics is decoherence. Generally, the mixed states and pure states will decohere when exposed to environment. Here, we will consider the general case of either a pure or a mixed system state exposed to environment. As we know, usually pure states will become mixed when interacting with environment, and the mixed states like pure states will lose purity to become more mixed than ever for the system–bath interaction; thus, we can use the evolution of purity of a state to estimate the decoherence under environment.

To some extent, decoherence can be considered a symbol to express the reduction of purity. Therefore, we use

$$S(t) = F(\text{Tr}(\rho_s^2(t))) \quad (9)$$

as a function of decoherence. Here the function S is a monotonic decreasing function of purity of the system state $\text{Tr}(\rho_s^2(t))$. One least complicated case [17] is

$$S(t) = 1 - \text{Tr}(\rho_s(t)^2) \quad (10)$$

which is usually called linear entropy. Linear entropy has been applied to measure both entanglement and decoherence for different perspectives. In the view of the specific form of linear entropy, it is a function of purity of state and the purity is a sound expression to symbolize the environment effect; so linear entropy is often used to measure decoherence. A similar study has been carried out on decoherence by using linear entropy in [21, 22]. In this paper, we will employ linear entropy to estimate the decoherence of the three-qubit system in a fermionic environment.

3. Main results

We consider a general three-qubit pure quantum state

$$|\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle \quad (11)$$

as the initial system state, where the parameters satisfy the relation for normalization $|a|^2 + |b|^2 + |c|^2 + |d|^2 + |e|^2 + |f|^2 + |g|^2 + |h|^2 = 1$. For mixed states, the following calculation still holds. Then the whole density matrix

$$\rho(0) = \rho_s(0) \otimes \rho_b = |\psi\rangle\langle\psi| \otimes \rho_b. \quad (12)$$

For the commutation relation between H_s and $H_{sb} + H_b$, the evolution of the whole density matrix is

$$\begin{aligned} \rho(t) &= \frac{1}{Z} \left[e^{-it(H_s+H_{sb}+H_b^{\text{mf}})} \rho_s(0) e^{-H_b^{\text{mf}}/T} e^{it(H_s+H_{sb}+H_b^{\text{mf}})} \right] \\ &= \frac{e^{-m^2 J N}}{Z} \exp \left\{ it \sum_k \left[\left(\frac{J_0}{\sqrt{N}} (S_A^z + S_B^z + S_C^z) + 2mJ \right) S_k^z + \omega S_k^x \right] \right\} \end{aligned}$$

$$\begin{aligned} & \times \rho'_s \exp \left\{ (1/T) \left(\omega \sum_k S_k^x + 2Jm \sum_k S_k^z \right) \right\} \\ & \times \exp \left\{ -it \sum_k \left[\left(\frac{J_0}{\sqrt{N}} (S_A^z + S_B^z + S_C^z) + 2mJ \right) S_k^z + \omega S_k^x \right] \right\} \end{aligned} \quad (13)$$

where $\rho'_s = e^{it\xi_0(S_A^z S_B^z + S_B^z S_C^z + S_A^z S_C^z)} \rho_s(0) e^{-it\xi_0(S_A^z S_B^z + S_B^z S_C^z + S_A^z S_C^z)}$. The reduced system matrix can be obtained by tracing out the degree of freedom of the bath

$$\rho_s(t) = \text{Tr}_b(\rho(t)). \quad (14)$$

Now, our main task is to give an analytical result for $\rho_s(t)$. Before we give the final reduced matrix of the system, we first introduce some formulae to simplify the calculation. Firstly, we take Q as the following form:

$$Q = (q_1 \sigma_x + q_2 \sigma_z) = \begin{pmatrix} q_2 & q_1 \\ q_1 & -q_2 \end{pmatrix} \quad (15)$$

with q_1, q_2 real coefficients. The exponentiation of Q gives

$$e^Q = (\cosh q)I + \frac{\sinh q}{q} Q; \quad e^{iQ} = (\cos q)I + i \frac{\sin q}{q} Q \quad (16)$$

where $q = \sqrt{q_1^2 + q_2^2}$ and I is the identity operator with rank 2. Similarly, the partition function Z will take the following value as the Hamiltonian of the thermal bath H_b^{mf} in equation (6) has a similar form to Q in equation (15):

$$Z = e^{-m^2 J N/T} \left\{ \left(2 \cosh \frac{\Theta}{2T} \right)^N \right\}. \quad (17)$$

Secondly, we apply the formulae of equations (15) and (16) to equation (14) to calculate the reduced system matrix as shown in equation (14), and obtain the following expression:

$$\prod_k \text{Tr}_b \{ e^{iI'} e^R e^{i\tilde{I}} \} = 2^N \left(\cosh \frac{\Theta}{2T} \right)^N \left(\cos \left(\frac{(u-v)tmJ J_0}{2\Theta\sqrt{N}} \right) + i \frac{\Theta}{J} \sin \left(\frac{(u-v)tmJ J_0}{2\Theta\sqrt{N}} \right) \right)^N. \quad (18)$$

Note that we get equation (18) under the approximation of neglecting the $O(\frac{1}{N})$ for N large. In equation (18) u and v are results of left and right spin operators of the system acting on the different elements of the system density matrix respectively in equation (14). Since the system density matrix can be expanded by a set of bases, we can calculate the time evolution of the basis used to expand the system density matrix based on the structure of the Hamiltonian form to get the final resulting system density matrix. For example, we take one basis as $|010\rangle\langle 110|$; then

$$\begin{aligned} & \left[\left(\frac{J_0}{\sqrt{N}} (S_{01}^z + S_{02}^z + S_{03}^z) + 2mJ \right) S_k^z + \omega S_k^x \right] \\ & \times |010\rangle\langle 110| \left[\left(\frac{J_0}{\sqrt{N}} (S_{01}^z + S_{02}^z + S_{03}^z) + 2mJ \right) S_k^z + \omega S_k^x \right] \\ & = \left[\left(\frac{J_0}{2\sqrt{N}} + 2mJ \right) S_k^z + \omega S_k^x \right] |010\rangle\langle 110| \left[\left(\frac{-J_0}{2\sqrt{N}} + 2mJ \right) S_k^z + \omega S_k^x \right]. \end{aligned} \quad (19)$$

Now in equation (19) $u = 1$ and $v = -1$. Here I' , R and \tilde{T} in equation (18) take the following forms:

$$I' = t \left[\left(\frac{uJ_0}{2\sqrt{N}} + 2mJ \right) S_k^z + \omega S_k^x \right] \tag{20}$$

$$R = (\omega S_k^x + 2mJ S_k^z) / T \tag{21}$$

$$\tilde{T} = -t \left[\left(\frac{vJ_0}{2\sqrt{N}} + 2mJ \right) S_k^z + \omega S_k^x \right]. \tag{22}$$

Finally, we get the reduced density matrix $\rho_s(t)$,

$$\begin{pmatrix} |a|^2 & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} & ah^*A_6 \\ M_{21} & |b|^2 & bc^* & bd^*A_2 & be^* & bf^*A_2 & bg^*A_2 & M_{28} \\ M_{31} & cb^* & |c|^2 & cd^*A_2 & ce^* & cf^*A_2 & cg^*A_2 & M_{38} \\ M_{41} & db^*A_2^* & dc^*A_2^* & |d|^2 & de^*A_2^* & df^* & dg^* & M_{48} \\ M_{51} & eb^* & ec^* & ed^*A_2 & |e|^2 & ef^*A_2 & eg^*A_2 & M_{58} \\ M_{61} & fb^*A_2^* & fc^*A_2^* & fd^* & fe^*A_2^* & |f|^2 & fg^* & M_{68} \\ M_{71} & gb^*A_2^* & gc^*A_2^* & gd^* & ge^*A_2^* & gf^* & |g|^2 & M_{78} \\ ha^*A_6^* & M_{82} & M_{83} & M_{84} & M_{85} & M_{86} & M_{87} & |h|^2 \end{pmatrix}. \tag{23}$$

For convenience, we use M_{ij} to represent the corresponding element of the density matrix, where

$$\begin{aligned} M_{12} = M_{21}^* &= ab^* e^{i(\xi_0 t)} A_2, & M_{13} = M_{31}^* &= ac^* e^{i(\xi_0 t)} A_2, \\ M_{14} = M_{41}^* &= ad^* e^{i(\xi_0 t)} A_4, & M_{15} = M_{51}^* &= ae^* e^{i(\xi_0 t)} A_2, \\ M_{16} = M_{61}^* &= af^* e^{i(\xi_0 t)} A_4, & M_{17} = M_{71}^* &= ag^* e^{i(\xi_0 t)} A_4 \\ M_{28} = M_{82}^* &= bh^* e^{-i(\xi_0 t)} A_4, & M_{38} = M_{83}^* &= ch^* e^{-i(\xi_0 t)} A_4, \\ M_{48} = M_{84}^* &= dh^* e^{-i(\xi_0 t)} A_2, & M_{58} = M_{85}^* &= eh^* e^{-i(\xi_0 t)} A_4, \\ M_{68} = M_{86}^* &= fh^* e^{-i(\xi_0 t)} A_2, & M_{78} = M_{87}^* &= gh^* e^{-i(\xi_0 t)} A_2 \end{aligned}$$

$$A_{u-v} = \left(\cos \left(\frac{(u-v)tmJJ_0}{2\Theta\sqrt{N}} \right) + i \frac{\Theta}{J} \sin \left(\frac{(u-v)tmJJ_0}{2\Theta\sqrt{N}} \right) \right)^N \tag{24}$$

and $*$ represents complex conjugation. In the limit of large N we can approximate it as

$$|A_{u-v}| \approx \exp \left(-\frac{(u-v)^2 t^2 J_0^2 m^2}{8} \left(\frac{J^2}{\Theta^2} - 1 \right) \right) \tag{25}$$

where u and v are defined as in equations (18)–(22). So far, we have obtained the final time-dependent density matrix.

3.1. Entanglement evolution

In practice, we are interested in some explicit cases which are of promising use in QIP. We will study the six computable functions to quantify the entanglement of the final states resulted from initial states, either pure quantum states or mixed ones.

Case 1. Obviously, we find that the diagonal states of the three-qubit do not change during the interaction with the thermal bath, therefore the *negativities* of ρ_{diag} are zero forever, where ‘diag’ means diagonal. The diagonal states we consider here belong to the class states whose density matrices are diagonal and positive matrices with unit trace. For example, the density

matrix $\rho_s(0) = \frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|)$ is one diagonal state. For any diagonal state in the fermionic environment we consider here, we get $\rho_{\text{diag}}(t) = \rho_{\text{diag}}(0)$ and

$$N(\rho_{\text{diag}}(t)) = N(\rho_{\text{diag}}(0)) = 0. \quad (26)$$

Here the *negativities* include the six computable functions. If we expose diagonal states to the environment, they will remain still, and so do their entanglement and decoherence.

Case 2. Let us set the parameters in equation (11), $a = h = \frac{1}{\sqrt{2}}$, other parameters are zero, that is the GHZ state (note that all GHZ states in this paper are the same) $|\psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ as the initial state. When exposed to the environment, the initial state evolves into the resultant state $\rho_{\text{GHZ}}(t)$. After calculation, we get

$$N_{(\text{AB})-\text{C}}(\rho_{\text{GHZ}}(t)) = \frac{1}{2}|A_6| \quad (27)$$

where $|A_6|$ is $|A_{u-v}|$, when $u - v$ equals 6 (note that following $|A_4|$ and $|A_2|$ are of analogous meaning). Obviously, the entanglement of the GHZ state shows an exponential decay with time as the entanglement of initial state is $\frac{1}{2}$, i.e. $N_{(\text{AB})-\text{C}}(\rho_{\text{GHZ}}(0)) = \frac{1}{2}$. Similarly, we can know that both $N_{(\text{BC})-\text{A}}(\rho_{\text{GHZ}}(t))$ and $N_{(\text{AC})-\text{B}}(\rho_{\text{GHZ}}(t))$ are the same as $N_{(\text{AB})-\text{C}}(\rho_{\text{GHZ}}(t))$ for symmetry. For the reduced density of two parties, $N_{\text{A-B};\text{C}}$ is zero and the other two are the same. When time goes to infinity, any of the *negativities* we calculated here will be zero, and we get the final state $\rho_{\text{GHZ}}(t \rightarrow \infty) = (\frac{1}{2}|000\rangle\langle 000| + |111\rangle\langle 111|)$ which obviously possesses no entanglement.

Case 3. If we choose the W state $|\psi\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$ as the initial state, that is the state in equation (11) with $b = c = e = \frac{1}{\sqrt{3}}$ and other parameters zero, it is easy to find that the environment has no effect on the entanglement of the W state ρ_{Wp} . We get the entanglement

$$N_{(\text{AB})-\text{C}}(\rho_{\text{Wp}}(t)) = N_{(\text{AB})-\text{C}}(\rho_{\text{Wp}}(0)) = 0.4714 \quad (28)$$

$$N_{\text{A-B};\text{C}}(\rho_{\text{Wp}}(t)) = N_{\text{A-B};\text{C}}(\rho_{\text{Wp}}(0)) = 0.206. \quad (29)$$

The other four *negativities* have corresponding values for symmetry. Actually, we can obtain such a result from the reduced system matrix in equation (23) straightforwardly because the W state does not perceive the symmetry-broken environment. To have a second look at the reduced system density matrix, we have no difficulty in finding that the W class states have the same property as the one singled out here. W class states will be explained later in subsection 3.3. On the basis of the result, we can say that the W state is strong under the environment, so the W state is the appropriate candidate for the information carrier under the system–bath interaction.

Case 4. Given $a = g = \frac{1}{\sqrt{2}}$, the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + b|11\rangle) \otimes |0\rangle$ with the corresponding density matrix is a partially separable state as the party C is separable from the two parties (AB). The effective density matrix of (AB), $\rho_{\text{AB}} = \text{Tr}_{\text{C}}(\rho_{\text{ABC}})$, may still retain some of the original entanglement. The negativity of ρ_{AB} , $N_{\text{A-B};\text{C}}$, can be used to quantify the residual entanglement. The residual entanglement of the state quoted here is given as $N_{\text{A-B};\text{C}}(\rho_{\text{AB}}(0)) = \frac{1}{2}$. The residual entanglement of the state will evolve as described in the following equation when the state is exposed to the environment,

$$N_{\text{A-B};\text{C}}(\rho_{\text{AB}}(t)) = \frac{1}{2}|A_4|. \quad (30)$$

Obviously, the entanglement decreases exponentially to 0 where the state density matrix is a diagonal one.

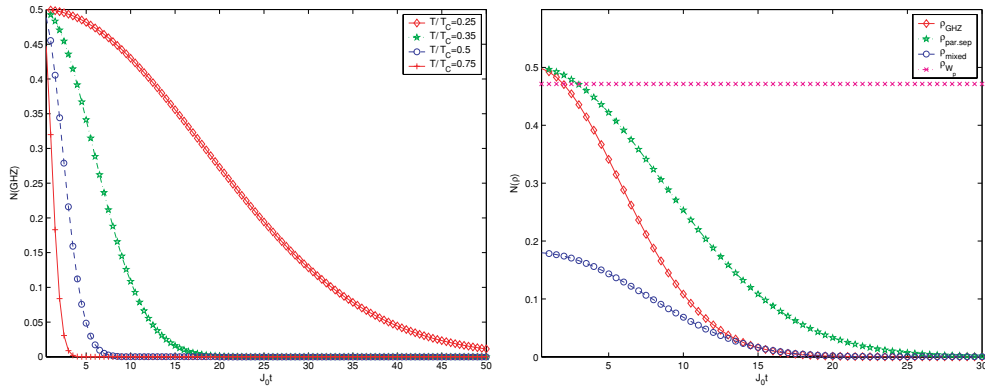


Figure 1. Entanglement versus scaled time $J_0 t$. The figure on the left describes the entanglement evolution of the GHZ state at different temperatures of the bath. While the entanglement evolutions of different states at a fixed temperature of $T/T_C = 0.35$ are shown in the figure on the right, $\omega = 0.1$ and $J = 2$.

Case 5. Having given some examples of pure states, we should study the entanglement evolution of a mixed state. Here, a mixed state

$$\rho_{\text{ABC}} = \frac{1}{2}|000\rangle\langle 000| + \frac{1}{6}|010\rangle\langle 010| + \frac{1}{4}(|000\rangle\langle 011| + |011\rangle\langle 000|) + \frac{1}{3}|011\rangle\langle 011| \quad (31)$$

with negativities $N_{(\text{AB})-\text{C}}(\rho_{\text{ABC}}(0)) = N_{(\text{AC})-\text{B}}(\rho_{\text{ABC}}(0)) = 0.1802$ and $N_{(\text{BC})-\text{A}}(\rho_{\text{ABC}}) = 0$. The entanglement evolved as described in the following equation

$$N_{(\text{AB})-\text{C}}(\rho_{\text{ABC}}(t)) = N_{\text{B}-\text{C};\text{A}}(\rho_{\text{BC}}(t)) = \frac{1}{12}(\sqrt{1 + 9|A_4|^2} - 1) \quad (32)$$

and $N_{(\text{AC})-\text{B}}(\rho_{\text{ABC}}(t))$ are the same as $N_{(\text{AB})-\text{C}}(\rho_{\text{ABC}}(t))$, where $\rho_{\text{BC}}(t) = \text{Tr}_{\text{A}}(\rho_{\text{ABC}}(t))$. The entanglement of the mixed state also decreases under the system–bath interaction. The final state of such an evolution will be a completely separable state.

From the above five cases, we find that, except for the W state, the other quantum entangled states, either pure or mixed, here will lose entanglement when interacting with the environment stated in this paper. We also find that the rates of entanglement loss are different for different states; in the case of the three-qubit system, the entanglement of the GHZ state gradually decreases faster than that of other states (see figure 1).

3.2. Decoherence

In this paper, we will use the linear entropy $S(t) = 1 - \text{Tr}(\rho_s^2(t))$ to obtain the decoherence for the system. In fact, we can estimate the decoherence of any state in principle. For simplicity, we just pick out some examples.

Case 1. Let the initial state be the GHZ state; we can have the decoherence which can be expressed as a function of $S(t)$,

$$S(t) = \frac{1}{2}(1 - |A_6|^2) \quad (33)$$

From equation (33), we find that the GHZ state will decohere when exposed to the environment. Let $t = 0$; then $S(0) = 0$, which indicates that the GHZ state gradually evolves from the pure

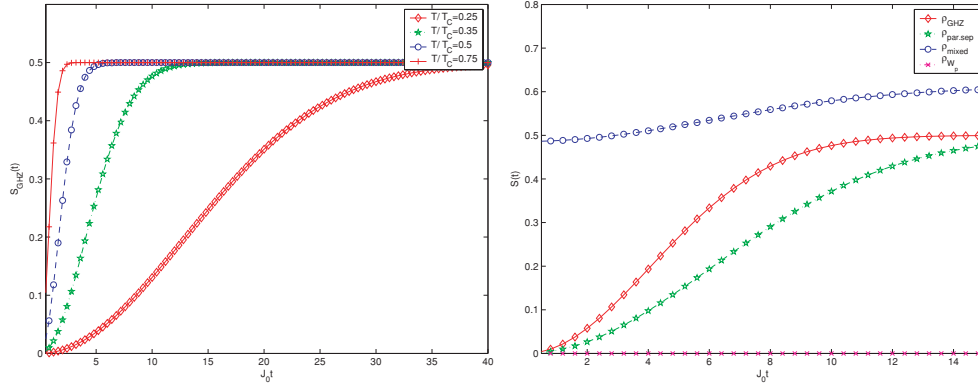


Figure 2. $S(t)$ versus the scaled time $J_0 t$. The figures on the left and right describe the evolution of $S(t)$ of the GHZ state at different temperatures and of different states at a fixed temperature $T/T_C = 0.35$ respectively. The values of the parameters are $\omega = 0.1$, $J = 2$.

quantum state into a mixed state. When $t \rightarrow \infty$, $S(t) = \frac{1}{2}$, where the final state is mixed as $\frac{1}{2}(|000\rangle\langle 000| + |111\rangle\langle 111|)$.

Case 2. Given the initial state is the W state, we obtain

$$S(t) = S(0) = 0. \quad (34)$$

Equation (34) suggests that the TIM environment has no effect on the coherence of the W state. Due to the same state structure, the W class states do not perceive the presence of the environment either. Therefore, the W class states are more robust than other states here under such a system–bath interaction.

Case 3. We pick out $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |0\rangle$ as the initial state,

$$S(t) = \frac{1}{2}(1 - |A_4|^2). \quad (35)$$

It is easy to find that the pure state will turn into a mixed state if the interaction with environment lasts for a long time. The initial state will become a diagonal state as $\frac{1}{2}(|000\rangle\langle 000| + |110\rangle\langle 110|)$ when t is infinite.

Case 4. Here, we will consider the decoherence of a mixed state which is the same as the above one used to study entanglement dynamics, that is $\rho_{ABC} = \frac{1}{2}|000\rangle\langle 000| + \frac{1}{6}|010\rangle\langle 010| + \frac{1}{4}(|000\rangle\langle 011| + |011\rangle\langle 000|) + \frac{1}{3}|011\rangle\langle 011|$. After a simple calculation, the function to measure decoherence is

$$S(t) = \frac{11}{18} - \frac{1}{8}|A_4|^2. \quad (36)$$

And the final state is $\rho_{ABC}(t \rightarrow \infty) = \frac{1}{2}|000\rangle\langle 000| + \frac{1}{6}|010\rangle\langle 010| + \frac{1}{3}|011\rangle\langle 011|$. Obviously, the environment makes the mixed state lose the coherence term.

Decoherence induced by the symmetry-broken environment has been illustrated in the above four cases. Except for the W class states and diagonal states, any other state will decohere under the interaction between the system and thermal bath. While the speeds of decoherence loss are different for different states, in the present three-qubit system, the GHZ state decoheres faster than any other state (see figure 2).

3.3. Further discussion

From the results of subsections 3.1 and 3.2, it is easy to find that the W state does not perceive the environment, and we give a second thought that the W class states have the same property as the W state. Here, we will further discuss the robustness of the W class states and other states under the environment.

In the case of the three-qubit system, any state can be expanded by a set of bases of E as

$$\rho_s = \sum_{\alpha, \beta=0}^7 c_{\alpha\beta} E_{\alpha\beta} \quad (37)$$

where $E_{\alpha\beta} = |\alpha_A \alpha_B \alpha_C\rangle \langle \beta_A \beta_B \beta_C|$, $\alpha_A, \alpha_B, \alpha_C, \beta_A, \beta_B, \beta_C$ take values 0 or 1 and $c_{\alpha\beta}$ is constant. For example, $E_{07} = |000\rangle \langle 111|$. Thus we call a class of states ρ_W as W class states if the class of states can be expanded by the basis $E_W = \{E_{11}, E_{22}, \dots, E_{66}, E_{12}, E_{21}, E_{14}, E_{41}, E_{24}, E_{42}, E_{35}, E_{53}, E_{36}, E_{63}, E_{56}, E_{65}\}$, that is

$$\rho_W = \sum_{\gamma, \delta} w_{\gamma\delta} E_{\gamma\delta} \quad (38)$$

where $w_{\gamma\delta}$ is constant, and $E_{\gamma\delta} \in E_W$. Let us consider one basis $E_{35}(0) = |011\rangle \langle 101|$ and the time evolution

$$\begin{aligned} E_{35}(t) &= \text{Tr}_b(e^{-iHt} E_{35}(0) \otimes \rho_b e^{iHt}) \\ &= \text{Tr}_b(e^{-i(H_{sb}+H_b)t} |011\rangle \langle 101| \otimes \rho_b e^{i(H_{sb}+H_b)t}) \\ &= \left[2 \cosh\left(\frac{\Theta}{2T}\right) \right]^{-N} \text{Tr}_b \left\{ \prod_k e^{i t \left[\frac{J_0}{\sqrt{N}} (S_A^z + S_B^z + S_C^z) + \omega S_k^x \right]} |011\rangle \langle 101| \right. \\ &\quad \left. \times e^{(\omega S_k^x + 2mJ S_k^z)/T} e^{-i t \left[\frac{J_0}{\sqrt{N}} (S_A^z + S_B^z + S_C^z) + \omega S_k^x \right]} \right\} \\ &= |011\rangle \langle 101| \left[2 \cosh\left(\frac{\Theta}{2T}\right) \right]^{-N} \prod_k \text{Tr}\{e^{iI_\gamma} e^R e^{i\tilde{I}_\delta}\} \\ &= E_{35}(0) \left[2 \cosh\left(\frac{\Theta}{2T}\right) \right]^{-N} \prod_k \text{Tr}\{e^{iI_\gamma} e^R e^{i\tilde{I}_\delta}\} \end{aligned} \quad (39)$$

where

$$\begin{aligned} I_\gamma &= t \left[\left(\frac{-J_0}{2\sqrt{N}} + 2mJ \right) S_k^z + \omega S_k^x \right] \\ R &= (2mJ S_k^z + \omega S_k^x) / T \\ \tilde{I}_\delta &= -t \left[\left(\frac{-J_0}{2\sqrt{N}} + 2mJ \right) S_k^z + \omega S_k^x \right] \end{aligned} \quad (40)$$

In fact, the above quantities are special cases of equations (20)–(22), when $u = -1$ and $v = -1$. By using our previous result of equation (18), we get the following expression:

$$\prod_k \text{Tr}\{e^{iI_\gamma} e^R e^{i\tilde{I}_\delta}\} = \left[2 \cosh\left(\frac{\Theta}{2T}\right) \right]^N. \quad (41)$$

Combination of equations (39) and (41) will give the final result

$$E_{35}(t) = E_{35}(0). \quad (42)$$

Analogously, we can prove that any basis $E_{\gamma\delta}$ of E_W remains unchanged under the fermionic environment, i.e. if $E_{\gamma\delta} \in E_W$, then

$$E_{\gamma\delta}(t) = E_{\gamma\delta}(0). \quad (43)$$

Since the W class states can be expanded by the set of E_W , i.e.

$$\rho_W(0) = \sum_{\gamma\delta} w_{\gamma\delta} E_{\gamma\delta}(0), \quad (44)$$

the evolution of W class states can be expressed as

$$\rho_W(t) = \sum_{\gamma\delta} w_{\gamma\delta} E_{\gamma\delta}(t). \quad (45)$$

With equations (43)–(45), the evolution of W class states is obtained,

$$\rho_W(t) = \sum_{\gamma\delta} w_{\gamma\delta} E_{\gamma\delta}(0) = \rho_W(0). \quad (46)$$

So the W class states do not perceive the presence of the environment.

Similarly, we can prove that the environment has no effect on the basis E_{00} or E_{77} as it does on E_{35} . While the diagonal states ρ_{diag} can be expanded by the set of bases $E_{\text{diag}} = \{E_{00}, E_{11}, E_{22}, \dots, E_{77}\}$, thus we can prove that the environment has no effect on diagonal states either.

We apply a similar process to any basis $E_{\alpha\beta} \in E$ which does not belong to E_W or E_{diag} , and find that $E_{\alpha\beta}$ will decohere under the environment. So far we have got the class of decoherence-free states ρ_{DF} of our model, and ρ_{DF} can be expanded by the set of bases $E_{\text{DF}} = E_W \cup E_{\text{diag}}$. Of course, ρ_{DF} remains unchanged under the system–bath interaction. Any state which cannot be expanded by E_{DF} will decohere under the environment, and evolve into a resulting state belonging to ρ_{DF} after a long time.

4. Conclusion

Entanglement dynamics and decoherence of the three-qubit system in a symmetry-broken fermionic environment are studied. By using the mean field approximation, we obtain analytical results of entanglement evolution and decoherence by employing *negativity* and linear entropy respectively. The model considered in this paper is an extension of that in the work [15] from two-qubit to three-qubit, and it resembles an array of Rydberg atoms interacting with a cavity mode of radiation field [14]. In such instances, we can select three atoms as a system to represent the three-qubit system in our model.

In this paper, we have analysed the evolution of entanglement and coherence of the three-qubit system. We find that, except for the class of decoherence-free states ρ_{DF} , any other state will lose coherence under the symmetry-broken environment. The entanglement of any state belonging to the class states ρ_{DF} remains unchanged, while that of other states will decrease or remain constant due to the system–bath interaction. In the cases considered in this paper, we find that the GHZ state loses entanglement or coherence faster than the other states do. We also find that the speeds of disentanglement (loss of entanglement) and decoherence depend on temperature too. From the figures, the lower the value of T/T_C , the longer time the states remain entangled.

In conclusion, we believe our analysis is helpful for a better understanding of entanglement dynamics and decoherence.

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